Ch.6 Arrays

Geometry of *N*-Element Linear Array

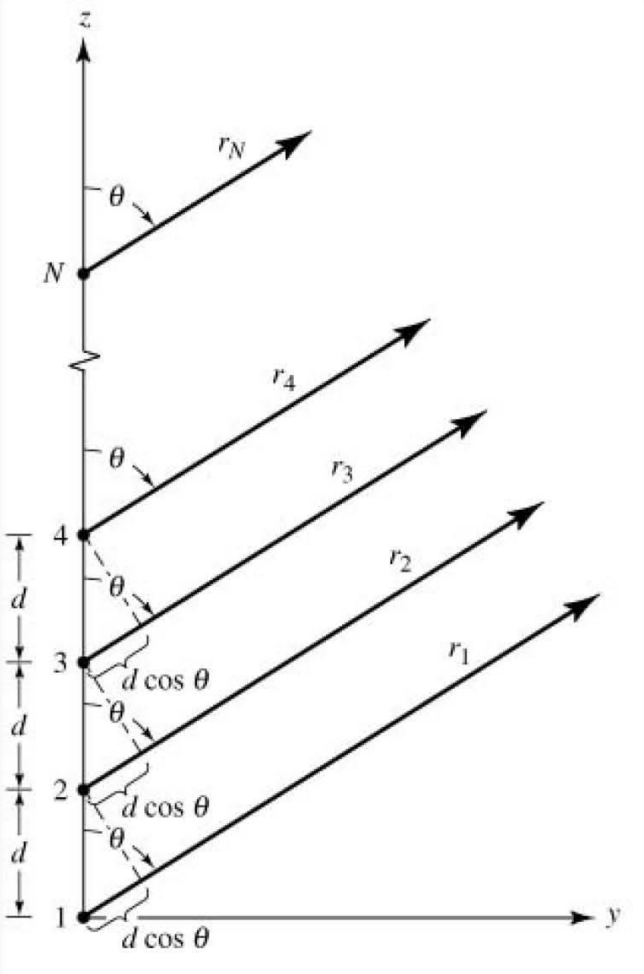


Fig. 6.5(a)

N-Elements

$$AF = \underbrace{1e^{j0}}_{\#1} + \underbrace{1e^{j\beta}e}_{\#2}^{jkd\cos\theta} + \underbrace{1e^{j2\beta}e}_{\#3}^{j2kd\cos\theta}$$

$$+ \cdots + \underbrace{1e^{j(N-1)\beta}e}_{\#N}^{j(N-1)kd\cos\theta}$$

$$AF = \sum_{n=1}^{N}(1)e^{j(n-1)\beta}e^{j(n-1)kd\cos\theta}$$

(6-6)

N-Elements

$$AF = \sum_{n=1}^{N} e^{j(n-1)\left(\underbrace{kd\cos\theta+\beta}_{\psi}\right)} \quad (6\text{-}6)$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (6\text{-}7)$$

$$\psi = kd\cos\theta + \beta \quad (6\text{-}7a)$$

Another Form Of *AF*

$\boxed{1}$: $AF = 1 + e^{j\psi} + e^{j2\psi} + ..$

$$..+ e^{j(N-2)\psi} + e^{j(N-1)\psi}$$

$\boxed{2}$: $e^{j\psi} AF = e^{j\psi} + e^{j2\psi} + ..$

$$..+ e^{j(N-2)\psi} + e^{j(N-1)\psi} + e^{jN\psi}$$

$\boxed{2} - \boxed{1}$:

$$AF\left(-1+e^{j\psi}\right) = -1 + e^{jN\psi} \qquad (6\text{-}9)$$

Another Form Of *AF*

$$AF\left(-1+e^{j\psi}\right)=-1+e^{jN\psi} \qquad (6\text{-}9)$$

$$AF=\frac{e^{jN\psi}-1}{e^{j\psi}-1}=\frac{e^{j\frac{N}{2}\psi}\left(e^{j\frac{N\psi}{2}}-e^{-j\frac{N\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{j\frac{\psi}{2}}-e^{-j\frac{\psi}{2}}\right)}$$

Divide Numerator and Denominator by 2j

$$AF = e^{j\frac{\psi}{2}(N-1)} \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)} \qquad (6\text{-}10)$$

$$|AF| = \left|e^{j\frac{\psi}{2}(N-1)}\right| \left|\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

$$|AF| = \left|\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right| \qquad (6\text{-}10a)$$

$$AF = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)} \overset{\psi\to 0}{\cong} \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}} \qquad (6\text{-}10b)$$

Normalized by N:

$$(AF)_n = \frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)} \overset{\psi\to 0}{\cong} \frac{\sin\left(\frac{N\psi}{2}\right)}{\frac{N\psi}{2}} \qquad (6\text{-}10c,d)$$

$$\psi = kd\cos\theta + \beta$$

Nulls

$$(AF)_n = \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)} \simeq \frac{\sin\left(\frac{N\psi}{2}\right)}{\frac{N\psi}{2}} = 0$$

$$\sin\left(\frac{N\psi}{2}\right) = 0$$

$$\frac{N\psi}{2} = \sin^{-1}(0) = \pm n\pi, \quad n = 1, \cancel{2}, \cdots$$

$$n \neq N, 2N, \cdots$$

$$\frac{N}{2}(kd\cos\theta_n + \beta) = \pm n\pi$$

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\right)\right] \qquad (6\text{-}11)$$

Maxima (Principal)

$$AF = \frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{\psi}{2}\right)} \simeq \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 1$$

$$\sin\left(\frac{\psi}{2}\right) = 0 \Rightarrow \frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi$$

$$m = 0, 1, 2, \cdots$$

$$\psi = \pm 2m\pi = kd\cos\theta_m + \beta$$

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right], \quad m = 0, 1, 2 \qquad (6\text{-}12)$$

$$\underline{m = 0}: \quad \theta_m = \cos^{-1}\left(-\frac{\lambda\beta}{2\pi d}\right) \qquad (6\text{-}13)$$

Secondary Maxima

$$\frac{N}{2}\psi \simeq \pm\left(\frac{2s+1}{2}\right)\pi, \qquad s = 1, 2, 3, \ldots$$

$$\frac{N}{2}\left(kd\cos\theta_s + \beta\right) = \pm\left(\frac{2s+1}{2}\right)\pi$$

$$\theta_s \simeq \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm \left(\frac{2s+1}{N}\right)\pi\right]\right\} \quad \text{(6-15)}$$

$\underline{s = 1}$: $$\theta_s \simeq \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{3\pi}{N}\right)\right]$$

First Secondary Maxima

$$\frac{N}{2}\psi = \cdot \simeq \frac{3\pi}{2} = 4.7124 \qquad (6\text{-}16)$$

$$|AF| = \left|\frac{\sin(3\pi/2)}{3\pi/2}\right| = \frac{2}{3\pi} = 0.212 = -\underline{13.46\ dB}$$

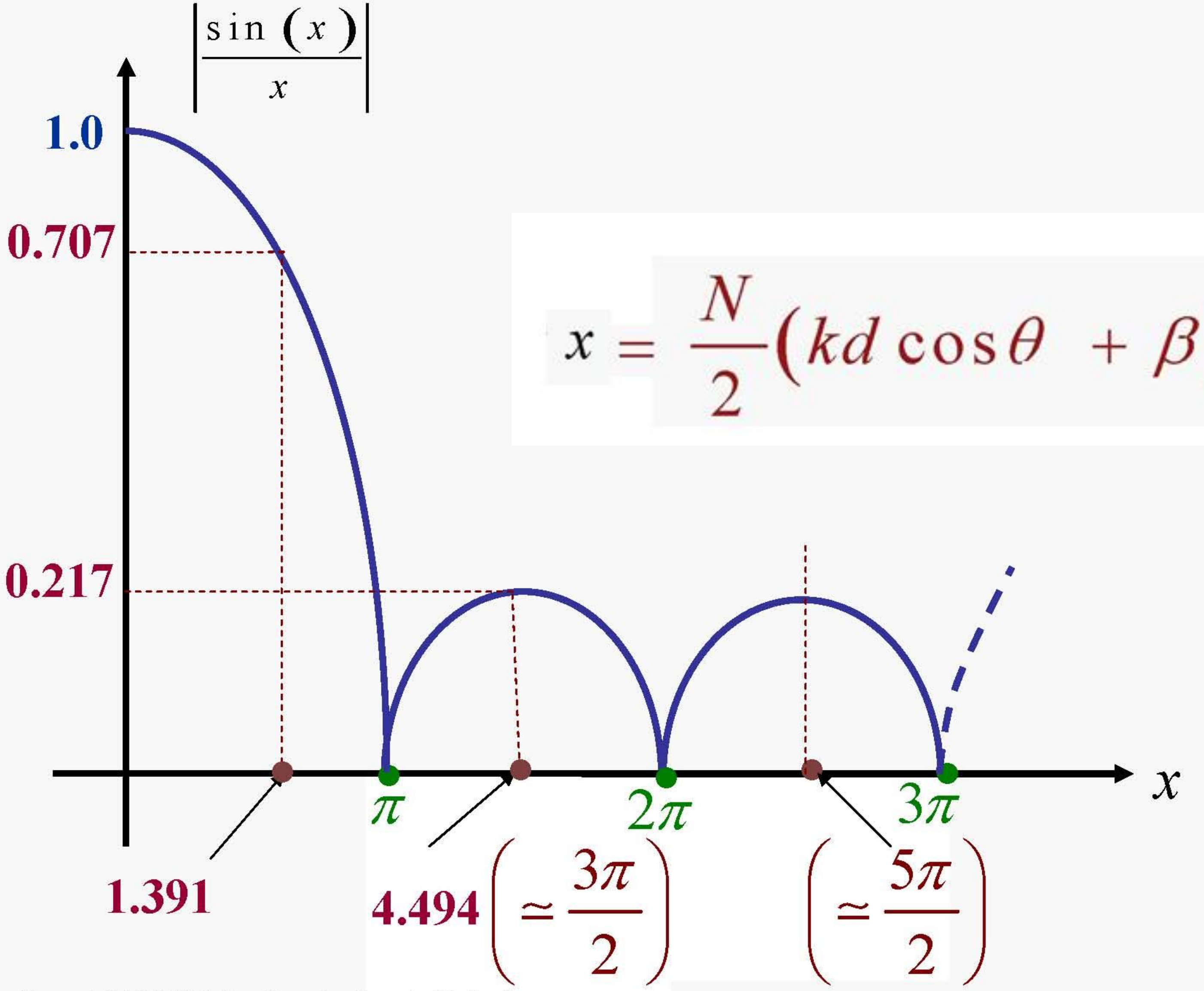
Half-Power (3-dB)

$$AF \simeq \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$$

$$\frac{N}{2}\psi = \frac{N}{2}\left(kd\cos\theta_h + \beta\right) = \pm 1.391$$

$$\theta_h \simeq \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right] \qquad (6\text{-}14)$$

$$\theta_h \simeq \frac{\pi}{2} - \sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right] \qquad (6\text{-}14a)$$



$\left|\frac{\sin(x)}{x}\right|$
1.0
0.707
0.217
$x = \frac{N}{2}(kd\cos\theta + \beta)$
π
2π
3π
x
1.391
4.494 $\left(\simeq \frac{3\pi}{2}\right)$
$\left(\simeq \frac{5\pi}{2}\right)$

Linear Arrays

1. Broadside

   $(\theta_m = 90^\circ)$

2. Ordinary End-Fire

   $(\theta_m = 0^\circ, 180^\circ)$

3. Phased (Scanning)

   $(0^\circ \le \theta_m \le 180^\circ)$

4. Hansen-Woodyard End-fire

   $(\theta_m = 0^\circ, 180^\circ)$

Broadside Array

$$\psi\Big|_{\theta=90^\circ} = (kd\cos\theta + \beta)\Big|_{\theta=90^\circ} = 0 + \beta = 0 \qquad \text{(6-18a)}$$

$$\boxed{\beta = 0}$$

Avoid $\underline{d = n\lambda}$ because

$$\psi\Big|_{\substack{\beta=0\\ d=n\lambda}} = (kd\cos\theta + \beta)_{\substack{\beta=0\\ d=n\lambda}} = 2\pi n\cos\theta$$

$$\psi = 2\pi n\cos\theta\Big|_{\theta=0^\circ,180} = \pm\, 2\pi n \qquad \text{(6-19)}$$

Broadside ($d = \lambda / 4$) $N = 10$

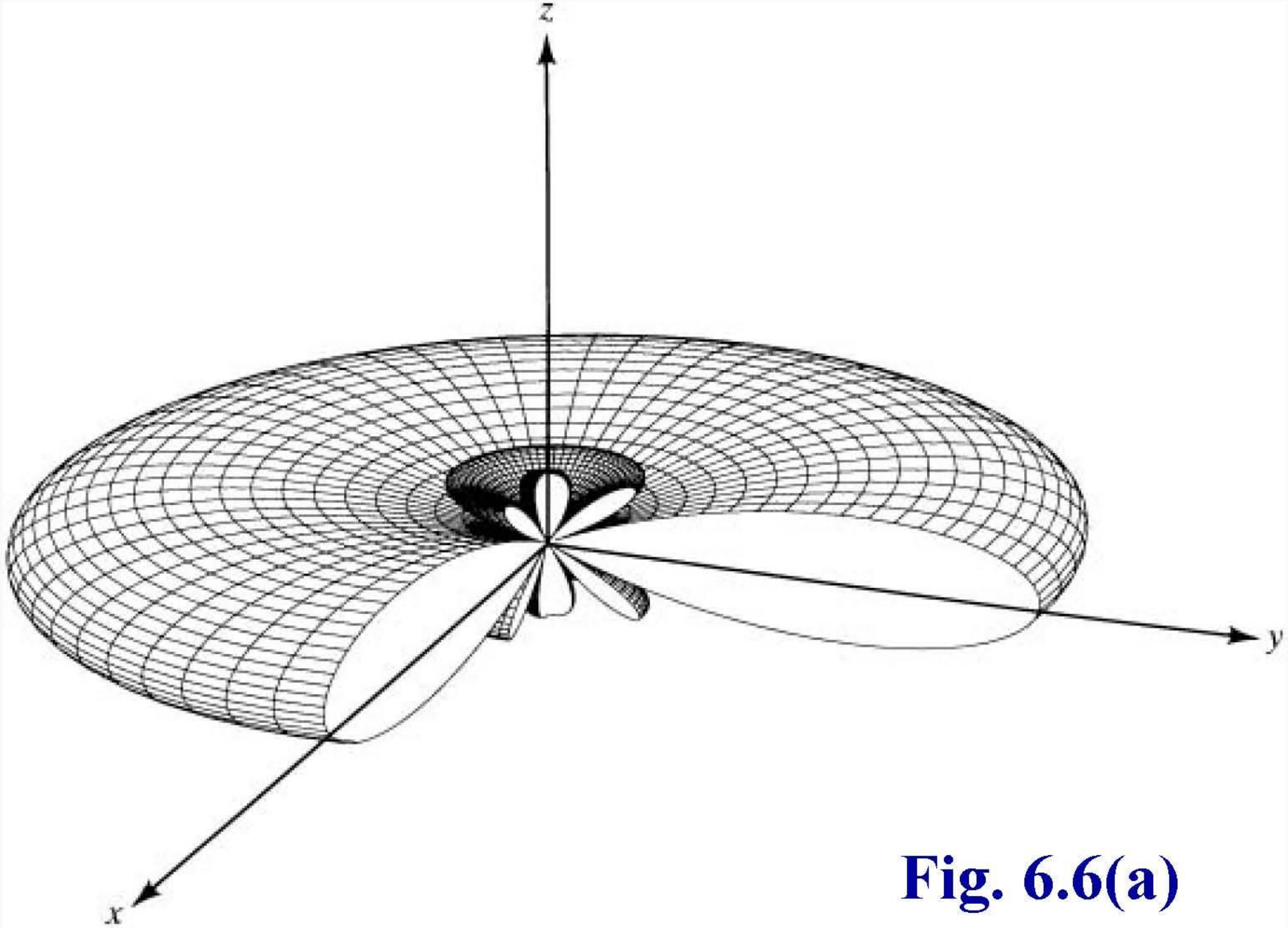


Fig. 6.6(a)

Broadside/End-Fire ($d = \lambda, N = 10$)

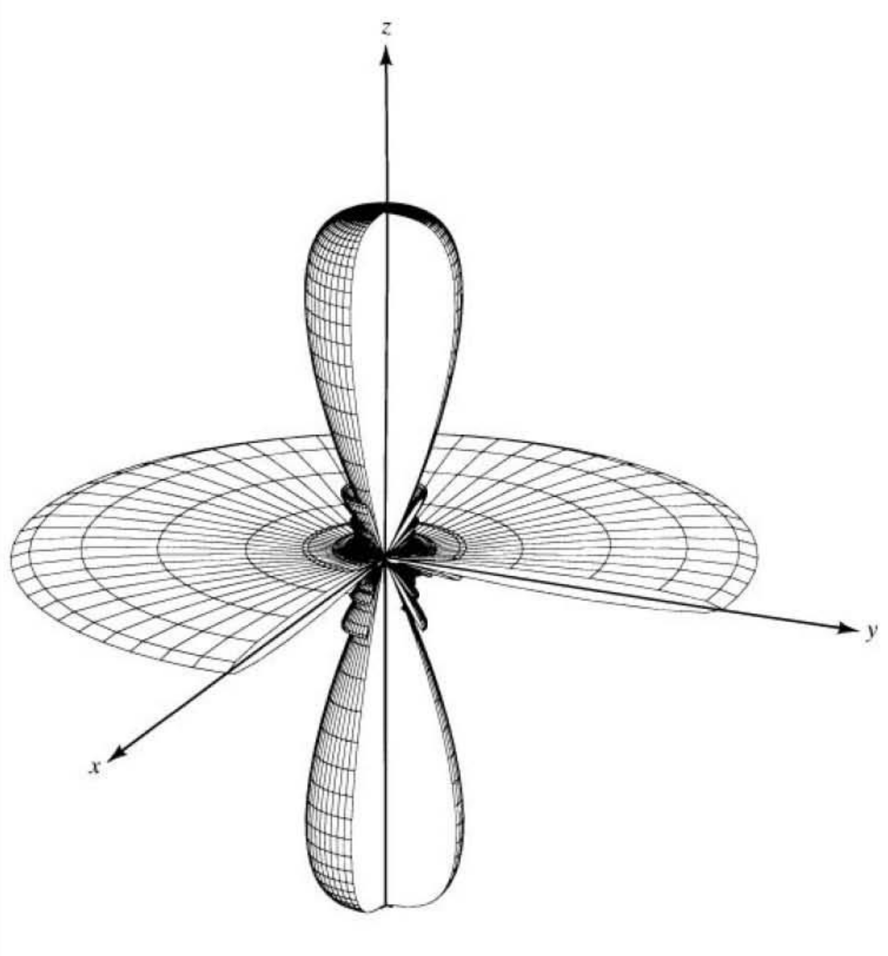


Fig. 6.6(b)

No Grating Lobes

$$d < \lambda$$

Grating Lobes

$$d \geq \lambda$$

10-Element Uniform Amplitude Broadside Array

$N = 10$

$\beta = 0$

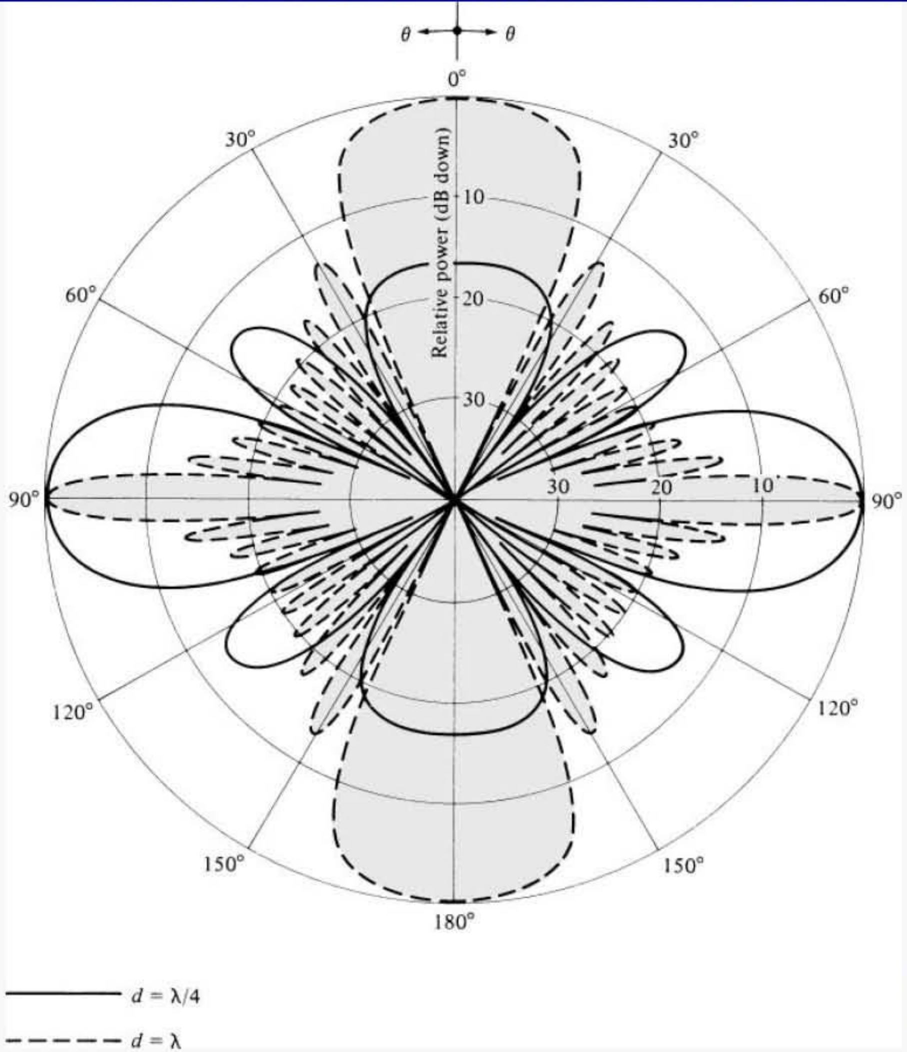


Fig. 6.7

Table 6.1 NULLS, MAXIMA, HALF-POWER POINTS, AND MINOR LOBE MAXIMA FOR UNIFORM AMPLITUDE BROADSIDE ARRAYS

NULLS	$\theta_n = \cos^{-1}\left(\pm \frac{n}{N}\frac{\lambda}{d}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(\pm \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(\pm \frac{1.391\lambda}{\pi N d}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[\pm \frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$ $s = 1, 2, 3, \ldots$ $\pi d/\lambda \ll 1$

Table 6.2 BEAMWIDTHS FOR UNIFORM AMPLITUDE BROADSIDE ARRAYS

FIRST NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi Nd}\right)\right]$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2dN}\right)\right]$ $\pi d/\lambda \ll 1$

Ordinary End-Fire

A. Maximum Toward θ=0°

B. Maximum Toward θ=180°

A. Toward θ=0°

$$\psi\Big|_{\theta=0^\circ} = (kd\cos\theta + \beta)\Big|_{\theta=0^\circ} = kd + \beta = 0$$

$$\boxed{\beta = -kd}$$

B. Toward θ=180°

$$\psi = (kd\cos\theta + \beta)\Big|_{\theta=180^\circ} = -kd + \beta = 0$$

$$\boxed{\beta = +kd}$$

Array Factor Patterns of a 10-Element Uniform Amplitude End-Fire Array (N=10,d=λ/4)

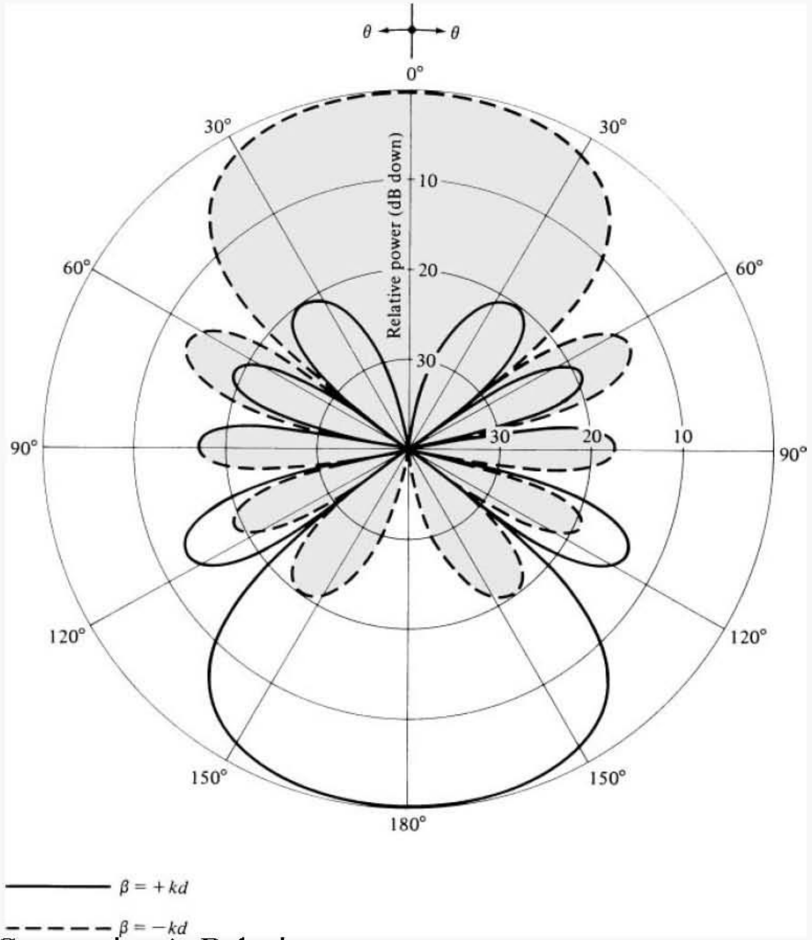


Fig. 6.9

Table 6.3 NULLS, MAXIMA, HALF-POWER POINTS, AND MINOR LOBE MAXIMA FOR UNIFORM AMPLITUDE ORDINARY END-FIRE ARRAYS

NULLS	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi dN}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$ $s = 1, 2, 3, \ldots$ $\pi d/\lambda \ll 1$

Table 6.4 BEAMWIDTHS FOR UNIFORM AMPLITUDE ORDINARY END-FIRE ARRAYS

FIRST NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\cos^{-1}\left(1 - \frac{3\lambda}{2Nd}\right)$ $\pi d/\lambda \ll 1$